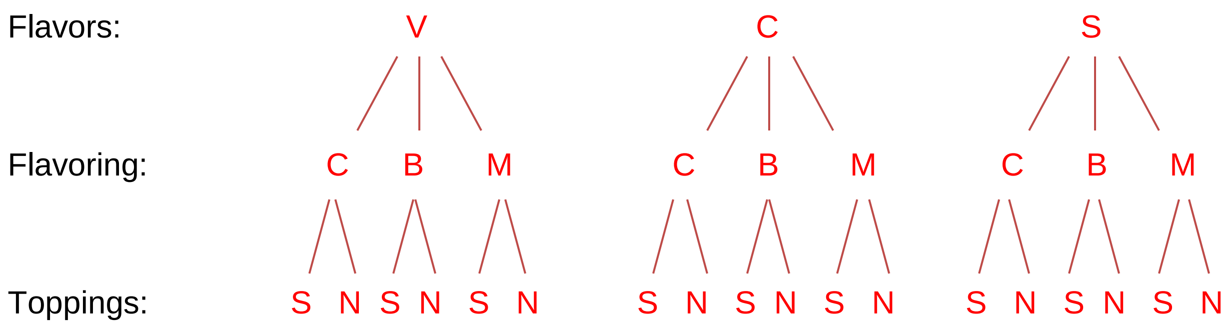
Counting Methods

For probability problems, we need to know how many outcomes are possible for complicated situations.

Ex 1. You go into an ice cream shop.
They have 3 flavors of ice cream (vanilla, chocolate and strawberry),
3 types of flavoring (cherry, butterscotch, mint chocolate),
and 2 extra toppings (sprinkles and nuts).
How many different types of sundaes can you make from these?

Solution 1: Make a tree diagram



So there are 18 total.

Solution 2: Use the Fundamental Counting Principle,

which says: "if you have an event which can occur m ways followed by another event which can occur n ways, the number of ways that the two events can occur is m times n".

If the outcome of the second event does not depend on what happened with the first event

the events are called independent events.

If the outcome of the 2nd event DOES depend on the 1st event, the events are dependent.

Are these events independent or dependent?

Pick an ice cream then pick a topping? independent

Pick a president and then a secretary from a group of students? dependent

Pick a card from a deck, put it back and then pick another? independent

Pic a card, leave it out, then pick another? dependent

Pick a card from a deck, then roll a die? independent

Roll a die, then roll it again? independent

Ex 2. The five players on a basketball team line up to shake hands with their opponents. How many ways can they line up?

Solution: How many choices are there for the first person? 5
How many choices are there for the second person? 4
How many choices are there for the third person? 3
How many choices are there for the fourth person? 2
How many choices for the last person? 1
So from the fundamental counting principle, the total number of ways is $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$ → n! is called "n factorial" and $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

A permutation is a way of arranging people or things where order DOES matter.

Ex 3: Now suppose you have a club with ten people in it. You need to elect a president and a vice president. How many ways can you pick these two people?

Order still matters for president and vice president, but not for the other 8 people.

$_nP_r$ = The number of permutations of "n things taken r at a time" = $\frac{n!}{(n-r)!}$

For this problem, n = 10 and r = 2, so $_{10}P_2 = \frac{10!}{2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 10 \cdot 9 = 90$

Ex 4. Now suppose you want to send two people (from your club of ten) out for snacks.

In this case, it doesn't matter the order in which the two people are picked.

A combination is a way of arranging people or thins where order DOES NOT matter.

$_nC_r$ = The number of combinations of "n things taken r at a time" = $\frac{n!}{(n-r)! \cdot r!}$

For this problem, n = 10 and r = 2, so $_{10}C_2 = \frac{10!}{8! \cdot 2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(2 \cdot 1)} = \frac{10 \cdot 9}{2} = 45$

Ex 5. Fifteen people are waiting in line for Taylor Swift's autograph. She agrees to sing autographs for 4 people. How many ways could this happen?

Is this a permutation or a combination? combination

The answer is: $_{15}C_4 = \frac{15!}{11! \cdot 4!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1} = 15 \cdot 7 \cdot 13 = 1365$

Ex 6. 8 people are running the 50 yard dash. How many ways can people finish 1^{st}, 2^{nd}, and 3^{rd}?

Is this a permutation or a combination? permutation

The answer is: ${}_{8}P_{3}$ $= \frac{8!}{3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720$